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Closed string tachyon condensation on twisted circles

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Abstract: We study IIA/B string theory compactified on twisted circles. These models possess closed string tachyons and reduce to type-0B/A theory in a special limit. Using methods of gauged linear sigma models and mirror symmetry we construct a conformal field theory which interpolates between these models and flat space via an auxiliary Liouville direction. Interpreting motion in the Liouville direction as renormalization group flow, we argue that the end point of tachyon condensation in all these models (including 0B/A theory) is supersymmetric type-II theory. We also find a zero-slope limit of these models which is best described in a T-dual picture as a type-II NS-NS fluxbrane. In this limit tachyon condensation is an interesting and well posed problem in supergravity. We explicitly determine the tachyon as a fluctuation of supergravity fields, and perform a rudimentary numerical analysis of the relevant flows.

Keywords: Sigma Models, Superstrings and Heterotic Strings, Superstring Vacua, Conformal Field Models in String Theory.

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1. Introduction

Worldsheet conformal field theories are solutions to the equations of motion of weakly coupled string theory. Conformal field theories that possess no relevant operators are stable solutions; small excitations about the corresponding spacetime background have positive energy. On the other hand, conformal field theories possessing relevant operators are tachyonic or unstable solutions of string theory.[1] It is of great interest to follow the tachyon condensation processes about such backgrounds, and to determine their endpoints.

There has been tremendous progress in the understanding of open string tachyon condensation, following the work of Sen [5]. Open string tachyons appear in non-supersymmetric configurations of branes, and the dynamics of tachyon condensation has been studied using a variety of techniques, among them string field theory, conformal field theory, and noncommutative geometry. The picture that emerges from this study is simple and physically satisfying. For instance, the tachyonic mode that appears when a D-brane is brought next to an antibrane is a signal of the tendency of these objects to annihilate. The endpoint of this tachyon condensation is simply empty space.

An open string tachyon is a boundary field on the worldsheet of the string. In the $g_{\rm s} \to 0$ limit, open string tachyon condensation modifies boundary conditions on the string worldsheet, but leaves the bulk conformal field theory untouched. In contrast, a zero-momentum closed string tachyon is a relevant deformation of the string sigma model. Consequently, closed string tachyon condensation modifies the bulk CFT and changes the spacetime on which the string propagates. Thus the dynamics of closed string tachyon condensation may be expected to be more interesting, but more complicated, than in the open string case.

Nonetheless Adams, Polchinski, and Silverstein [6] have recently argued for a pattern of closed string tachyon condensation about certain non-supersymmetric orbifolds that qualitatively resembles that of open string tachyon condensation. They argue, for example, that C/Z_n orbifolds of type-II theory are finite energy, unstable excitations of type-II theory in flat space, to which they decay in the process of tachyon condensation. This decay proceeds via an expanding bubble of flat space embedded in a static sea of C/Z_n. Very recently, Vafa [7] has elegantly argued for the same picture by studying flows in gauged linear sigma models and their mirror counterparts (see also [8, 9, 10, 11]).

The process of tachyon condensation seems even more mysterious in theories that are not easily interpreted as finite excitations of supersymmetric backgrounds, as there are no clear candidates for the endpoint of this process. Type-0A/B strings [12, 13] are especially interesting tachyonic theories. These 10-dimensional Lorentz invariant theories can be obtained from the supersymmetric type-IIA/B string theories by a $(-1)^F$ orbifold. Unlike the (perhaps even more interesting) bosonic string, type-0 strings possess worldsheet supersymmetry and have R-R antisymmetric tensor fields and D-branes carrying R-R charges. The endpoint of tachyon condensation in these theories is of great interest. Recently this question was given an intriguing twist in [14] where a duality was proposed between type-0A string theory and a Kaluza-Klein-Melvin fluxbrane. This duality suggests that type-0A

[1] For early work on tachyon condensation in string theories see [1, 2, 3, 4].

theory is an excited state of IIA theory (see [15, 16] for similar conjectures in heterotic theories), and motivated the authors of [17] to conjecture that the endpoint of type-0A tachyon condensation is the supersymmetric type-IIA vacuum. Unfortunately, the duality of [14] is nonperturbative and so cannot be easily used to verify the conjecture of of [17].

Rather than attempting a frontal assault on the problem of type-0 tachyon condensation, in this paper we will try to outflank it by studying a large class of type-II backgrounds that includes the type-0 string theories as a special limit. We will find it useful to study the so-called "interpolating orbifold," in which type-II strings are put on a circle of radius R with antiperiodic (Scherk-Schwarz) boundary conditions for the spacetime fermions.[2] In the limit $R \to \infty$, the boundary conditions become unimportant and the theory returns to type-II strings in flat space. In the limit $R \to 0$, on the other hand, the theory goes over to type-0 strings in flat space (0B/A if we started with IIA/B) [20]. We will review this limit in section 2.1.

Even more room to maneuver can be had by introducing a second parameter that interpolates between normal and Scherk-Schwarz boundary conditions. Anti-periodicity for the fermions can be imposed by accompanying the circle identification with a 2π rotation in any orthogonal plane. An obvious generalization is therefore to choose an orthogonal plane, and then to accompany the circle translation by a rotation through an arbitrary angle $2\pi\zeta$. The resulting geometry is flat ten-dimensional space ($\vec{x} \in \mathbb{R}^{6+1}, y \in \mathbb{R}, z \in \mathbb{C}$), quotiented by the isometry

$$(\vec{x}, y, z) \sim (\vec{x}, y + 2\pi R, e^{2\pi i\zeta} z)\,. \tag{1.1}$$

We will refer to such a space (or the non-trivial three-dimensional part of it) as a "twisted circle." Without loss of generality we assume $0 \le \zeta \le 1$, with $\zeta = 0$ corresponding to a normal circle and $\zeta = 1$ to a Scherk-Schwarz one.

The twisted circles form a moduli space of string backgrounds, parametrized by R, ζ, and the constant value Φ_0 of the dilaton. (Due to the supersymmetry breaking at non-zero ζ, this is only a moduli space at string tree level.) Tseytlin and Russo have extensively studied string propagation on these backgrounds [21, 22, 23, 24].[3] In particular, they find that the spectrum of the type-II superstring includes a tachyon if and only if

$$R^2 < 2\alpha'\zeta. \tag{1.2}$$

We review this spectrum in section 2.2.

According to (1.2), there is a large range of parameters for which type-II theories on twisted circles are perturbatively unstable and decay via tachyon condensation. In this paper we apply two methods to the problem of finding the endpoint of this condensation.

In section 3, using the techniques of [7, 28, 29], we construct a worldsheet conformal field theory that may be interpreted as a supersymmetric sigma model with a four real dimensional target space.[4] One of the directions in target space is effectively a Liouville

[2] A similar strategy was used to study the non-supersymmetric heterotic strings in [18, 19].

[3] See also [25]. D-branes on twisted circles were discussed recently in [26, 27].

[4] See [30, 31] for related work.

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direction. At large positive values of the Liouville field, the three transverse directions of the target space reduce to a twisted circle geometry with a rational twist angle. At large negative values, on the other hand, they reduce simply to $\mathbb{R}^2 \times S^1$. Assuming that motion in the Liouville direction mimics the flow of the transverse three dimensional sigma model under renormalization group flow, we conclude that under the tachyonic perturbation it flows in the infrared to the supersymmetric sigma model with target space $\mathbb{R}^2 \times S^1$. Restated, we argue that the endpoint of tachyon condensation of a type-II theory compactified on a twisted circle (at least for rational values of the twist) is the same type-II theory compactified on an ordinary circle.

A different approach to the question of tachyon condensation is to find a zero-slope limit in which the tachyon is a mode of the supergravity fields. We describe such a limit in section 4. The twisted circle in this limit is most naturally described by T-dualizing, upon which it becomes an H-flux 7-brane [32, 33, 34, 24, 35]. The tachyon is a Gregory-Laflamme-like instability of the fluxbrane, which we find explicitly as a fluctuation of the NS-NS supergravity fields. In this limit the problem of tachyon condensation is an interesting and well posed problem in supergravity and so may (for instance) be simulated on a computer. The study of tachyon condensation about H-flux branes, consequently, constitutes a check of the general arguments we present in section 3. We set up this problem, and briefly report the inconclusive results of our rudimentary attempts at a numerical analysis of the relevant equations.

In section 5 we end with a discussion of our results and their implications. In appendices A and B, we supply some technical details on the tachyon and on RG flows. In appendices C and D we study the worldsheet conformal field theory of strings on twisted circles. We determine the tachyon vertex operator and compute its four point function. In appendix E we use the results of appendix D to analytically study the initial dynamics of tachyon condensation when a fluxbrane is compactified on a circle whose radius is chosen to make the tachyon almost massless.

2. Review of type-II strings on twisted circles

2.1 0B/A as IIA/B on a vanishing Scherk-Schwarz circle

Consider type-IIA strings on a Scherk-Schwarz circle of radius R. In the limit $R \to 0$, all the states of non-zero momentum around the circle are lifted out of the spectrum. This includes all the fermions, since by the antiperiodic boundary conditions they have half-integral momenta. In the zero-momentum sector, the effect of the boundary conditions is to reverse the GSO projection for states of odd winding number w (this follows from the modular invariance of the 1-loop partition function [20]). Thus when $w = 2w'$ the spectrum consists of the (NS+,NS+) and (R+,R−) sectors, and for $w = 2w' + 1$ the (NS−,NS−) and (R−,R+) sectors. This is approximately the spectrum in the zero-momentum, winding number w' sector of type-0A theory on a circle of radius $2R$. This equality becomes exact in the limit $R \to 0$, where it is more appropriate to describe the theory as 0B in infinite flat space. It is similarly straightforward to show that not only the spectrum but also the OPEs of the two theories are the same in this limit. Hence we arrive at the statement that type-IIA compactified on a Scherk-Schwarz circle of zero size is the same as type 0B at infinite radius. By similar arguments, type IIB on a vanishing Scherk-Schwarz circle yields type 0A.

2.2 Spectrum of type-II strings on a twisted circle

Type-II strings propagating on twisted circles are described by a free worldsheet theory, and so are easily quantized. The spectrum of the resulting theory must be modified from that of type-II strings on an ordinary circle in several ways.

First, wave functions in the zero mode sector take the form $e^{im\phi+i\alpha y}$, where m is an integer as usual but $\alpha = (k - m\zeta)/R$; k is an integer (instead of the usual $\alpha = k/R$).

Next, note that the shortest winding number w string that obeys $|z| > r$ has squared length $L^2 = (2\pi R)^2 + (2\pi r\zeta)^2$. Thus the world sheet Hamiltonian includes a restoring harmonic oscillator potential in the z-plane, which confines all such states to the origin. In particular, the zero modes in the z-plane of all winding modes are lifted.

The lifting of the zero modes in the z-plane is a special case of a more general effect. In the winding number w sector, $z(\sigma,\tau)$ obeys the boundary conditions $z(2\pi,\tau) = e^{2\pi i\zeta} z(0,\tau)$, and so may be mode expanded as

$$z(\sigma,\tau) = i\sqrt{\alpha'}e^{i\zeta y_0}\left(\sum_{n+\zeta w\in Z}\beta_n e^{-in(\sigma+\tau)} + \sum_{n-\zeta w\in Z}\tilde{\beta}_n e^{in(\sigma-\tau)}\right). \tag{2.1}$$

The shift in the moding of the z oscillators results in a shift in their level number, which it is not difficult to check is proportional to the angular momentum:

$$N_{z,\mathrm{L}}(\zeta) = N_{z,\mathrm{L}}(0) + \zeta w J_{z,\mathrm{L}}\,, \qquad N_{z,\mathrm{R}}(\zeta) = N_{z,\mathrm{R}}(0) - \zeta w J_{z,\mathrm{R}}\,, \tag{2.2}$$

where $J_{z,\mathrm{L}}$ and $J_{z,\mathrm{R}}$ are the contribution of left and right movers, respectively, to the angular momentum in the z plane (recall that the angular momentum is the sum of occupation numbers of 'anticlockwise moving' modes minus the sum of occupation numbers of 'clockwise moving' modes). The change of moding in (2.1) also results in a modification of the zero-point energy in the NS sector.

Putting together these observations, we arrive at the following mass spectrum [21, 22, 23] in the NS-NS sector:

$$M^2 = \frac{2}{\alpha'}N_0 + \left(\frac{wR}{\alpha'}\right)^2 + \left(\frac{m - \zeta J_{z,\mathrm{L}} - \zeta J_{z,\mathrm{R}}}{R}\right)^2 - \frac{2\zeta w}{\alpha'}(J_{z,\mathrm{R}} - J_{z,\mathrm{L}} - 1)\,. \tag{2.3}$$

Here M is the mass relative to the remaining $6+1$-dimensional Lorentz symmetry, while N_0 is the total level number at $\zeta = 0$ (including $-1/2$ for each NS sector, to make it an integer). We note that (2.3) is only correct for $0 \le \zeta \le 1$. For instance, when $1 \le \zeta \le 2$, the formulae above remain valid if ζ is replaced by $\zeta' = 2 - \zeta$. We will use this fact in section 3 below.

It is instructive to consider the special case $\zeta = 1$. Since this involves compactification on a circle with a 2π twist, one might be tempted to conclude that the bosonic spectrum of this theory is identical to the spectrum of type-II theory compactified on an ordinary circle. However, the shift in the moding of the z oscillators shifts the worldsheet fermion numbers by $\pm w$ as ζ is taken from 0 to 1. This effectively reverses the GSO projection in odd winding sectors, as was asserted in the previous subsection.

It is evident from (2.3) that tachyons can only occur in the winding sectors, and (for $w > 0$) only if $J_{z,\mathrm{R}} - J_{z,\mathrm{L}} > 0$. Indeed, already at the lowest level allowed by the type-II GSO projections ($N_0 = 0$), there is an (NS+,NS+) state with $J_{z,\mathrm{R}} = 1$, $J_{z,\mathrm{L}} = -1$. In fact, this state, with $m = 0$ and $w = 1$, hence $M^2 = (R/\alpha')^2 - 2\zeta w/\alpha'$, is the first state to become tachyonic as we increase ζ. Consequently, a necessary and sufficient condition for the twisted circle to be tachyonic is $2\alpha'\zeta > R^2$. As we take ζ to 1, this state flows to an (NS-,NS-) state, and in the limit $R \to 0$ it becomes the zero-momentum bulk tachyon of the T-dual type-0 theory.

3. Tachyon condensation from a linear sigma model analysis

As reviewed above, type-II theories on twisted circles are tachyonic if $R^2 < 2\alpha'\zeta$. The addition of a zero-momentum tachyon to the world sheet conformal field theory induces a renormalization group flow of that theory. In this section we will study this flow, and argue that its endpoint is supersymmetric type-II theory compactified on an ordinary circle.

Our arguments will be rather indirect. In section 3.1 we will use an $\mathcal{N} = (2, 2)$ linear sigma model to construct a non-linear sigma model with a four (real) dimensional target space. One of the target dimensions is a Liouville direction; at large positive values of the Liouville field, the other three dimensions reduce to the free conformal field theory describing the twisted circle background, while at large negative values they reduce to the free theory on $C \times S^1$. We interpret motion in the Liouville direction as evolution under renormalization group flow of the three dimensional sigma model when perturbed by its tachyonic operator, thereby concluding that it flows in the IR to the flat $C \times S^1$ sigma model, restoring space-time supersymmetry.

3.1 The linear sigma model

Consider an $\mathcal{N} = (2, 2)$ U(1) gauged linear sigma model in two dimensions [36] containing, in addition to the gauge multiplet Σ, two charged chiral superfields Φ_1 and Φ_{-n} and an axionic chiral superfield P with the identification $P \sim P + 2\pi i$. Under U(1) gauge transformations the fields transform as

$$\Phi_1 \to e^{i\alpha}\Phi_1\,, \qquad \Phi_{-n} \to e^{-in\alpha}\Phi_{-n}\,, \qquad P \to P + i\alpha\,. \tag{3.1}$$

The action for this system[5] is

$$S = \frac{1}{2\pi}\int d^2\sigma d^4\theta \left[\bar{\Phi}_1 e^V \Phi_1 + \bar{\Phi}_{-n} e^{-nV}\Phi_{-n} + \frac{k}{4}(P + \bar{P} + V)^2 - \frac{1}{2e^2}|\Sigma|^2\right]. \tag{3.2}$$

[5]This system was analyzed in detail in a beautifully written paper [28]. We follow the notations of that paper in what follows.

For future reference we explicitly write the relevant parts of this action in component fields using the Wess-Zumino gauge. The pieces of the action involving the auxiliary gauge field D are

$$\frac{1}{2\pi}\int d^2\sigma\left(\frac{D^2}{2e^2}+D(|\varphi_1|^2-n|\varphi_{-n}|^2+kp_1)\right), \tag{3.3}$$

where φ_1, φ_{-n} and $p=p_1+ip_2$ refer to the lowest components of Φ_1, Φ_{-n} and P respectively. The kinetic terms for the scalar fields are

$$\frac{1}{2\pi}\int d^2\sigma(-\mathcal{D}^\mu\varphi_1\mathcal{D}_\mu\varphi_1-\mathcal{D}^\mu\varphi_{-n}\mathcal{D}_\mu\varphi_{-n}-\frac{k}{2}\mathcal{D}^\mu p\mathcal{D}_\mu p)\,, \tag{3.4}$$

where $\mathcal{D}_\mu\varphi_1$, $\mathcal{D}_\mu\varphi_{-n}$ are the usual covariant derivatives, while $\mathcal{D}_\mu p=\partial_\mu p+iv_\mu$ (v_μ is the gauge boson). It follows from the identification of P and the normalization of the kinetic term (3.4) that p_2 parameterizes a circle of radius $R=\sqrt{\alpha' k}$.

In (3.2), (3.3), and (3.4) we presented the classical action for our system. Quantum mechanically this action is renormalized. Supersymmetry ensures that the renormalization of the D-term is independent of e and so is a purely one loop effect [36]. The divergent contribution of one loop graphs to the D term is

$$\frac{1}{2\pi}\int d^2\sigma\,(D(1-n)\ln\Lambda)\,. \tag{3.5}$$

This divergent contribution may be absorbed into a renormalization of p_1:

$$p_1(\Lambda)=p_1(\mu)+\frac{n-1}{k}\ln\left(\frac{\Lambda}{\mu}\right). \tag{3.6}$$

Notice that $p_1(\Lambda)$ tends to ∞ in the UV and to $-\infty$ in the IR.

We wish to study the dynamics of this model at low energies. The gauge multiplet of the theory is always massive (with $m\geq e\sqrt{k}$) and may be integrated out. At sufficiently low energies we may restrict attention to fluctuations on the supersymmetric manifold of zero-energy configurations satisfying

$$|\varphi_1|^2-n|\varphi_{-n}|^2+kp_1=0\,, \tag{3.7}$$

modulo gauge transformations, so that the dynamics is captured by a two complex dimensional supersymmetric sigma model. In order to explicitly study this sigma model, we will use the gauge freedom and (3.7) to solve for one of the three fields $(\varphi_1,\varphi_{-n},p)$. We will then plug this solution into the kinetic terms (3.4) and integrate out the gauge boson v_μ. This procedure is classical; however as we will see below, in the limit $|p_1|\to\infty$ the gauge boson is infinitely massive. Since two dimensional gauge theories are free in the UV, when $|p_1|$ is sufficiently large the classical approximation is valid; we will carry out the analysis only in this limit.

When $p_1>0$, φ_{-n} cannot be zero (see (3.7)), so we fix the gauge by setting it real and positive. However, this gauge choice leaves unfixed a residual Z_n group of gauge transformations, generated by

$$\varphi_1\to e^{2\pi i/n}\varphi_1,\qquad p_2\to p_2+\frac{2\pi}{n}\,. \tag{3.8}$$

Eq. (3.7) may then be used to solve for φ_{-n}:

$$\varphi_{-n} = \sqrt{\frac{|\varphi_1|^2 + kp_1}{n}} = \sqrt{\frac{kp_1}{n}} + \mathcal{O}\left(\frac{1}{\sqrt{p_1}}\right). \tag{3.9}$$

As the "Higgs" field φ_{-n} has a vev of order $\sqrt{p_1}$, the v_μ gauge field is very massive for large p_1 and may now be classically integrated out of (3.4). In this limit $v_\mu = \mathcal{O}(1/|\varphi_{-n}|^2) = \mathcal{O}(1/p_1)$ and so may be set to zero at leading order. The kinetic term for $|\varphi_1|$ is also suppressed compared to that for p_1 by $\mathcal{O}(1/p_1)$. Hence the dynamics in this limit is governed by the flat sigma model

$$S = \frac{1}{2\pi}\int d^2\sigma\left(\partial_\mu\varphi_1\partial^\mu\bar{\varphi}_1 + \frac{k}{2}\partial_\mu p\partial^\mu\bar{p},\right). \tag{3.10}$$

where φ_1 and p are identified under the Z_n group (3.8). Consequently, φ_1 and p_2 parametrize a twisted circle of radius $R = \sqrt{\alpha' k}/n$ and twist angle (in type-II theories) $\zeta = 1+1/n$. (We will explain below why $\zeta = 1 + 1/n$ rather than $1/n$.)

When p_1 is large and negative, the analysis presented above may be carried through with only minor modifications. A non-singular choice of gauge, in this case, is to set φ_1 to be real and positive, which leaves no residual gauge group. Solving for φ_1 instead of φ_{-n}, repeating the analysis above yields the flat space sigma model

$$S = \frac{1}{2\pi}\int d^2\sigma\left(\partial_\mu\varphi_{-n}\partial^\mu\bar{\varphi}_{-n} + \frac{k}{2}\partial_\mu p\partial^\mu\bar{p}\right). \tag{3.11}$$

where φ_{-n} is an unconstrained chiral field and p_2 is periodic with period 2π. Consequently (3.11) describes a sigma model on $C \times S^1 \times R$ where the radius of the circle is $\sqrt{\alpha' k}$.

We now come to an important subtlety. The central charge of this low energy conformal field theory is not equal to 6, even though it reduces in some regions to a sigma model on a flat four dimensional target space. Recall that, under a change of scale, p_1 is additively renormalized (see (3.6)). This implies that, for large $|p_1|$, the conformal field theory of the canonically normalized free boson $X \equiv p_1\sqrt{k\alpha'}$ is governed by the linear dilaton system with charge $V = (1-n)/\sqrt{k\alpha'}$, and central charge $c = 1 + 6\alpha' V^2 = 1 + 6(n-1)^2/k$ (see eq. (2.5.2) of [37] for notation). Consequently, the net central charge for the CFT is $c = 6 + 6(n-1)^2/k$. This is in precise agreement with the analysis given in [28].

In summary, the linear sigma model flows at low energies to a $c = 6{+}6(n{-}1)^2/k$ conformal field theory [28]. For $p_1 \gg 0$, this CFT reduces to the sigma model Liouville$\times$(twisted circle). For $p_1 \ll 0$, it reduces to the sigma model Liouville$\times(C \times S^1)$.

3.2 Mirror description

We will now study the mirror [29] of the gauged linear sigma model defined in the previous section [28]. The mirror model is a Landau-Ginzburg model with two twisted chiral superfields U and Y_P, and superpotential (see [28, 7])

$$\tilde{W} = e^{-nU} + e^{-U - Y_P/n}. \tag{3.12}$$

In the low energy limit, this model is conformal, and so both of the operators in the superpotential are marginal.

According to the mirror map, $\mathrm{Re}\, Y_P \sim k\, \mathrm{Re}\, P$.[6] Consequently, in the asymptotic region $p_1 \gg 0$ the second term in the superpotential is negligible, and the Landau-Ginzburg model reduces to the twisted circle times Liouville model. The second operator is a marginal deformation of this theory, whose effect vanishes for large $\mathrm{Re}\, Y_P$. It can be decomposed into three factors: a twist operator e^{-U} in the plane; a winding operator $e^{-i\mathrm{Im}\, Y_P/n}$ on the circle; and a dressing by $e^{-\,\mathrm{Re}\, Y_P/n}$. Together the first two factors constitute a tachyon vertex operator on the twisted circle (1.1); the third is a dressing by the Liouville mode,[7] making it both marginal and $\mathcal{N}=2$ supersymmetric.

Before ending this subsection we address two subtleties that we have ignored so far. Firstly, the worldvolume theory of a type-II string is governed by a conformal field theory, modded out by a chiral GSO projection. As discussed in [6, 7, 11], it is possible to impose such a projection on the theory constructed above only if n is odd.[8] Further, this projection removes fields with even twists, retaining all odd twisted fields, in particular the twist one tachyon of (3.12). Secondly, the Z_n identification (3.8) could be interpreted as acting on φ_{-n} as a rotation of either $2\pi/n \times n$ or $2\pi(1+1/n)\times n$. Only the second of these actions acts trivially on spacetime fermions and is consistent with the chiral GSO projection. Consequently, in type-II theory, we are forced to interpret the $\mathrm{Re}\, P \to \infty$ region of the CFT above as a twisted space with twist $\zeta = 1 + 1/n$ as we did in section 3.1.

3.3 Renormalization group flow

In this subsection we will employ the CFT of section 3.1 in an argument to predict the endpoint of tachyon condensation in type-II theories on twisted circles with twist $\zeta = 1+1/n$ (equivalently $\zeta' = 1 - 1/n$ — see under (2.3)).

Consider the twisted circle CFT whose central charge is $\hat{c} = 3$, together with an independent CFT of central charge $\hat{c} = 6 - 4(n-1)^2/k$. (The role of this theory is merely to provide additional central charge to the system. It remains a spectator through the action described below.) Perturb the twisted circle CFT by the infinitesmal addition of a tachyon, and formally couple this full system to two dimensional supergravity. The Weyl mode of the two dimensional graviton is dynamical, and combines with the original system to form a conformal field theory with $\hat{c} = 10$.

We now study this $\hat{c} = 10$ conformal field theory. Firstly note that the Weyl field in this theory does not decouple from the perturbed twisted circle CFT, as the tachyonic perturbation in the latter receives a Weyl dressing that makes it marginal. However, as the Weyl field ω represents the scale factor of the twisted circle CFT, the effect of this perturbation vanishes for $\omega \to \infty$ (we choose notation such that $\omega \to \infty$ is the UV). In this region the theory reduces to the sum of a linear dilaton system for ω ($\hat{c} = 1 + 4(n-1)^2/k$) and the original conformal field theories, together with an increasingly unimportant dressed

[6] More precisely $\mathrm{Re}\, Y_P \sim k(\mathrm{Re}\, P + V)$. However, for $|p_1| \gg 1$ the gauge multiplet is very massive, and may be set to zero.

[7] The dynamics of Y_P is governed by a Liouville theory [28]. In the rest of this section we will often refer to $\mathrm{Re}\, Y_P$ as a Liouville direction

[8] Refs. [6, 7, 11] actually discuss this for C/Z_n but the arguments carry over to the CFT of this section.

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tachyonic perturbation. The system will be complicated for finite values of ω. Its behaviour at large negative values of ω determines the endpoint of the RG flow initiated by a tachyonic perturbation in the twisted circle CFT.

We now observe that the CFT of section 3.1 has (together with the spectator CFT) all the properties expected of the CFT described in the previous paragraph. The central charges of the two theories match by construction. $\mathrm{Re}\, Y_P$ has the correct conformal transformation properties to be identified with ω. And at large values of $\mathrm{Re}\, Y_P$ it reduces to the twisted circle CFT times a linear dilaton system, with a dressed tachyonic perturbation. We thus propose that the endpoint of the renormalization group flow initiated by the tachyon on the twisted circle CFT of radius R and twist $\zeta = 1 + 1/n$ is determined by the $\mathrm{Re}\, Y_P \to -\infty$ region of the CFT of subsection 3.1 and so is given by a CFT on a supersymmetric circle of radius nR.

Given the formal nature of the above considerations, the following observation might be useful. Suppose we are given an exact time dependent solution to the equations of motion of type-II string theory starting at $t = -\infty$ at the twisted circle theory, and evolving in time to the endpoint of tachyon condensation. Analytically continuing this solution in time (presuming that is allowed), yields a $\hat{c} = 4$ conformal field theory that interpolates between the twisted circle CFT and the endpoint of tachyon condensation. The CFT of section 3.1 is such a theory except that its central charge $\hat{c}$ is greater than 4. The additional central charge reflects the fact that the motion in the $\mathrm{Re}\, Y_P$ is flow in scale rather than time.[9]

We now return to an important detail of this interpretation. As we have seen, the CFT of the previous sections for large $\mathrm{Re}\, Y_P$ is the product of the tachyon vertex operator and a Liouville dressing that keeps the full operator marginal. Hence the dimension of the Liouville dressing is $\zeta' - R^2/(2\alpha')$.[10] If $R^2 > 2\alpha'\zeta'$, so that the "tachyon" is not actually tachyonic, then RG flow drives the surface on which the theory lives to larger values of $\mathrm{Re}\, Y_P$, i.e. back to twisted circle. However, if $R^2 < 2\alpha'\zeta'$, then RG flow drives the theory to smaller values of $\mathrm{Re}\, Y_P$. As discussed in section 3.1 this means that at the endpoint the twist has disappeared and we have a larger ordinary circle.

[9] Indeed the linear dilaton provides a 'dissipation' term, allowing the motion to progress smoothly to the endpoint of tachyon condensation. Evolution in time would conserve energy, and so would presumably be considerably more complicated. The relation between Liouville and time evolution was studied in [38, 39, 40, 41, 42].

[10] This is easily directly verified in the UV Liouville CFT. As we have not presented the action for Y_P we work in linear sigma model variables; recall that $\mathrm{Re}\, Y_P = k\, \mathrm{Re}\, P$ for large $\mathrm{Re}\, P$ according to the mirror map. The holomorphic scaling dimension of

$$e^{-kp_1/n} = e^{\sqrt{k}X/\sqrt{\alpha'}n}$$

(X is the canonically normalized field defined in section 3.1) is

$$-\frac{\alpha'}{2} V \frac{\sqrt{k}}{\sqrt{\alpha'}n} \frac{\alpha'}{4} \frac{k}{-\alpha' n^2} = \frac{n-1}{2n} - \frac{k}{4n^2}$$

where V is the slope of the linear dilaton (see section 4.1). Rewriting this expression in terms of the deficit angle and radius we find the scaling dimension $-R^2/4\alpha' + \zeta'/2$.

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3.4 Conclusions from this construction

In this section we have argued that the endpoint of tachyon condensation for type-II theory on a twisted circle with twist $\zeta = 1+1/n$ and radius R is the same theory compactified on an ordinary circle of radius nR. This result immediately leads to a prediction for the endpoint of tachyon condensation in type-0A/B theories. Recall that, for instance, uncompactified type-0B theory is type-IIA theory on a twisted circle with $\zeta = 1$ and $R \to 0$. In order to set $\zeta = 1$ we take n to infinity holding R fixed; the endpoint of tachyon condensation is IIA theory in uncompactified 10 dimensional space, for all values of R including $R \to 0$. Consequently, it would appear that the endpoint of 0B/A tachyon condensation is IIA/B theory in flat space!

Of course, taking this limit could involve subtleties, since the localized tachyon of the twisted circle becomes the delocalized tachyon of type-0 theory. Zamolodchikov c-theorem-type arguments suggest that an RG flow seeded by such a tachyon cannot end at a theory with the same central charge. Note, however, that we are not strictly speaking studying RG flow here, but rather “Liouville flow.” This type of evolution may more closely mimic dynamical evolution, particularly insofar as it leaves the central charge invariant.

A generalization of the arguments of this section would imply that the final endpoint of tachyon condensation in IIA/B theory on a twisted circle of radius R and with twist $\zeta = 1 + m/n$ is IIA/B theory on an ordinary circle of radius nR. Further, as any irrational number can be increasingly accurately approximated by rational numbers with increasing denominators, the endpoint of tachyon condensation of IIA/B theory on a twisted circle of irrational ζ would appear to be uncompactified IIA/B theory. Heuristically, tachyon condensation unwinds the twist; this is achieved by a finite unwinding for rational ζ but an infinite unwinding for irrational ζ.

3.5 Supporting results

The decay of $(C \times C)/Z_{nm}$ orbifolds was studied in [6, 7, 11]. Let Z_{nm} act by a rotation of $e^{2\pi i/m}$ on the first C factor, and by a rotation of $e^{2\pi i/mn}$ on the second factor. In the limit $n \to \infty$, the second plane reduces to a cone with infinitesimal opening angle, and so (apart from the singularity at the origin) may be thought of as a zero size cylinder (this was noted in [6] in a very similar context). As the identification of this orbifold links rotation around this cylinder to a twist about the first C, this space approximates twisted cylinder of zero radius, with twist $1/m$. According to [7], the endpoint of tachyon condensation in this background is $C \times (C/Z_n)$ where the Z_n factor acts by rotations $e^{2\pi i/n}$, i.e. approximately an ordinary cylinder (still of zero radius) times an untwisted complex plane. This is in accordance with our our expectations described above.

3.6 Open questions

RG flow versus time evolution. Following several recent studies [6, 7, 11, 10], in this paper we have addressed tachyon condensation by studying renormalization group flows on the world sheet of the string instead of the actual on-shell process of tachyon condensation. While experience has often shown (for instance in studies of open string

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tachyon condensation) that these two processes have the same endpoint, it would be very interesting to have a better understanding of the relationship between them.

This question is especially delicate when the endpoint of RG flow is a CFT with marginal deformations, as other solutions exist in the neighbourhood of any such configuration. We have shown that RG flow (or more precisely Liouville flow) starting from the twisted circle theory ends at flat space on the smallest circle obtained after untwisting the original circle. The size of this final circle (a modulus of the endpoint) is finite for rational values of the twist, but infinite when the twist is a generic real number. It is noteworthy that the final circle depends so crucially on the rationality or otherwise of the twist. It would be interesting to understand if this feature persists under time evolution.

Connection with energy. Tachyon condensation in non-gravitational systems is a process that lowers energy (after all the excess energy has escaped to infinity in the form of radiation). It is not clear to us how the energies of two systems with different asymptotic geometries should be compared, but it would certainly be very interesting to understand closed string tachyon condensation as a process of energy minimization. See [10, 11] for recent ideas in this direction.

The flow of the dilaton. We have studied the RG flow of the string sigma model ignoring the dilaton. The dilaton coupling to the string worlsheet is proportional to $\alpha' \int d^2\sigma \sqrt{g}\Phi(X)R$, where g is the worldsheet metric, $\Phi(X)$ is the dilaton, and R is the worldsheet curvature scalar. Consequently, the string sigma model at tree level is independent of the dilaton.

The dilaton does modify the energy momentum tensor of the world sheet theory, and so the change in X under scale transformations is given by

$$\delta X^\mu = -\alpha' \nabla^\mu \Phi \delta \ln \Lambda + \text{higher order}. \tag{3.13}$$

However, this effect may be absorbed into a Λ dependent coordinate (X^μ) redefinition (we will see this in detail, in the supergravity approximation, in section 4, see above eq. (4.8)). Consequently, the renormalization group flow of the conformal field theory may be decoupled from the dilaton and studied independently, as we have done in this section.

On the other hand the behavior of the dilaton under RG flow certainly depends on (in fact is determined by) the flow of the other fields in the conformal field theory (this is explicit in the supergravity approximation, as we will see in section 4.4). It would be very interesting to study the evolution of the dilaton induced by the RG flows of this paper as well as those described in [6, 7], and in particular to determine the constant value of the dilaton at the endpoint of tachyon condensation.

4. Fluxbranes and the supergravity limit

4.1 A zero-slope limit

In this section we will find a zero-slope limit of the models with twisted circles. This limit is constructed so that the tachyon has a finite mass as $\alpha' \to 0$ (of course generic stringy oscillators decouple in this limit).

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Consider the mass formula (2.3). In order that states with winding number $w \neq 0$ remain in the spectrum, R must be scaled to zero along with α'. We set $R = \alpha'/R'$ and hold R' fixed when taking α' to zero. In order that the last term in (2.3) remains finite, we must also scale ζ to zero; we set $\zeta = q\alpha'/R'$, and hold q fixed. All states that are retained in this limit then have $m = 0$ and $N_0 = 0$. The spectrum of modes that survive this limit (in the NS-NS sector) is[11]

$$M^2 = \left(\frac{w}{R'}\right)^2 + 2wq(s_L - s_R + l_L + l_R + 1) + q^2(s_L + s_R + l_L - l_R)^2 \,, \tag{4.2}$$

Here l_L and l_R are the non negative occupation numbers of the zero mode z oscillators, and s_L and s_R the spins in the z plane of the GSO projected NS-NS ground state $b^R_{-\frac{1}{2}} b^L_{-\frac{1}{2}}|0\rangle$. This spectrum includes a tachyon if $qR' \leq 2$ (the most tachyonic mode has $w = 1$, $s_L = -1$, $s_R = 1$). For example, setting $s_L = -1$, $s_R = 1$, $l_R = l_L = 0$, (4.2) reduces to

$$M^2 = k^2 - 2kq \,. \tag{4.3}$$

where $k = w/R'$ is the momentum of the mode after T-duality.

4.2 The H-flux tube

The twisted circle geometry may be T-dualized on circles of constant $\tilde{z} \equiv e^{-i\zeta y/R} z$ (1.1) to obtain an NS-NS fluxbrane. The limit described above, together with an appropriate scaling of the dilaton, i.e.

$$R \to 0 \,, \qquad \zeta \to 0 \,, \qquad e^{\Phi_0} \to 0 \,, \tag{4.4}$$

with

$$q \equiv \frac{\zeta}{R} \,, \qquad e^{\Phi'_0} \equiv \frac{\sqrt{\alpha'}}{R} e^{\Phi_0} \tag{4.5}$$

held fixed has a particularly natural description in this T-dual picture. It is a flux 6-brane smeared in the y direction

$$\begin{aligned} ds^2 &= d\vec{x}^2 + \frac{1}{1+q^2r^2} dy^2 + dr^2 + \frac{r^2}{1+q^2r^2} d\phi^2 \,, \\ e^{2\Phi} &= \frac{1}{1+q^2r^2} e^{2\Phi'_0} \,, \qquad B = \frac{qr^2}{1+q^2r^2} dy \wedge d\phi \,, \end{aligned} \tag{4.6}$$

where y is now the T-dual of the old y direction (note $y \equiv y + 2\pi R'$) and $\tilde{z} = re^{i\phi}$. (In the classification of [43], this is the $a = -1$, $p = 6$, $k = 1$ fluxbrane.) The r-ϕ plane has a cigar geometry with asymptotic radius $1/q$, and the curvatures and H field strength are concentrated near $r = 0$.[12]

[11] For example, setting $s_L = -1$, $s_R = 1$, $l_R = l_L = 0$, (4.2) reduces to

$$M^2 = k^2 - 2kq \,. \tag{4.1}$$

where $k = w/R'$ is the momentum of the mode in this T-dual picture.

[12] The geometry (4.6) is obtained after a T-duality in the limit (4.4), (4.5), without necessarily also taking $\alpha' \to 0$. If in addition $q^2\alpha'$ is taken to zero (the limit of section 4.1), supergravity is a good approximation.

To reiterate, the twisted circle models in the zero slope description of the previous subsection are reliably described in supergravity as NS-NS H-Flux branes. It is easy to verify that (4.6) obeys the supergravity equations of motion. Since α' has been taken to zero, the entire spectrum (4.2), including the tachyon, must be reproduced in type-II supergravity about (4.6).

It is easy to explicitly determine the tachyon as a fluctuation of supergravity fields. We present our derivation in appendix A; roughly the linear fluctuation of the fields corresponding to the tachyon may be determined by T-dualizing the polarization of the tachyon vertex operator. We find

$$\begin{aligned}
\delta ds^2 &= e^{iky-kqr^2/2}\left(\left(dr - \frac{iqr}{1+q^2r^2}dy\right)^2 + \left(\frac{r}{1+q^2r^2}d\phi\right)^2\right),\\
\delta B &= ie^{iky-kqr^2/2}\left(dr - \frac{iqr}{1+q^2r^2}dy\right)\wedge\frac{r}{1+q^2r^2}d\phi,\\
\delta\Phi &= \frac{e^{iky-kqr^2/2}}{2(1+q^2r^2)}\,.
\end{aligned}\tag{4.7}$$

(Eq. (4.7) must be combined with the fluctuation related by $y\to -y$ to obtain a real fluctuation of the fields.) That (4.7) is indeed a normal mode oscillation of the background (4.6) with mass given by (4.3) can be checked using the supergravity equations of motion. Note that the tachyon is confined to the region near $r=0$ by its y momentum.

The fact that tachyon condensation in twisted models in the limit of section 4.1 may be studied in supergravity is interesting, as it provides a method to test the conclusions of the previous section, in a special case, using a (perhaps numerical) supergravity analysis. From another viewpoint it suggests an unfortunately rather singular solution to the endpoint of tachyon condensation about IIA/B flux branes in supergravity.

In the next subsection we will briefly address the decay of fluxbranes in supergravity, and use supergravity intuition to suggest possible endpoints. We will then observe that the prediction of section 3.5. infact predicts a singular endpoint. In the subsequent subsection we will present a crude numerical analysis of the relevant supergravity flows.

4.3 Flux tube decay: generalities

This tachyon (4.7) about (4.6) has much in common with the Gregory-Laflamme instability of black strings [44].[13] Like the black string, (4.6) is translationally invariant in one direction (the y direction). Like our tachyon (4.7), the Gregory-Laflamme instability about a black string is a mode that carries momentum in this translationally invariant direction and so breaks this translational invariance. The spectrum of the Gregory-Laflamme tachyon as a function of the momentum in this direction is qualitatively very similar to (4.3). In particular, (as is the case with for (4.3)), the Gregory-Laflamme tachyon is positive for sufficiently large momentum. Consequently both our fluxbrane and the black string are stable when compactified on sufficiently small circles.

[13] Similar observations were made in [45].

We recall standard lore about the Gregory-Laflamme tachyon condensation. Until recently it had been presumed that the Gregory-Laflamme instability, triggered at a particular wavelength, would lead to an array of black holes, which would then by further instabilities coalesce into ever larger black holes. However it was recently argued in [46] that such a motion is not possible (the argument invoked mild assumptions). The authors of [46] argue that the black string instead settles down into a stable (and as yet unknown) "string of pearls" configuration, at a preferred wavelength set by its Schwarzschild radius.

The analogy between (4.6) and the black string might lead one to explore the corresponding two endpoints to the decay of (4.6). The first is the SO(3)-symmetric NS flux 6-brane of [17, 47, 48], which carries an infinite amount of flux and has a conical geometry at infinity. The second is as yet unknown periodic arrangement of flux in the y-direction, which would not change the boundary conditions at infinity.

The analysis of the previous section, however, predicts a third and rather singular endpoint to this evolution. Before T-dualizing, the fluxbrane in IIA/B is described by the limit (4.4) together with $\alpha' \to 0$, i.e. $R = \alpha'/R'$, $\zeta = q\alpha'/R'$ with $\alpha' \to 0$ and q, R' fixed. As the denominator of $1-\zeta$ for small ζ is greater than or equal to $1/\zeta$, according to section 3.5 the endpoint of tachyon condensation is IIA/B theory on a circle of size at least $1/q$. T-duality relates this to IIB/A theory on a circle of size smaller than $\alpha' q = 0$ in the limit under consideration. The results of section 3.5 thus predict that the supergravity solution (4.6) will evolves into IIB/A on a zero size circle, i.e. a singular geometry. It is of great interest to verify this prediction directly in supergravity.

4.4 Flux tube decay: numerics

In our primitive attempts at a numerical simulation of the supergravity equations, we did not try to model the dynamical time evolution, but instead attempted to simulate worldsheet RG flow. The RG equations on the NS-NS fields are:

$$\begin{aligned} \frac{\dot{G}_{\mu\nu}}{\alpha'} &= -R_{\mu\nu} - 2\nabla_\mu\partial_\nu\Phi + \frac{1}{4}H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu\,, \\ \frac{\dot{B}_{\mu\nu}}{\alpha'} &= \frac{1}{2}\nabla^\alpha H_{\alpha\mu\nu} - H_{\alpha\mu\nu}\partial^\alpha\Phi + H_{\mu\nu\rho}\xi^\rho, \\ \frac{\dot{\Phi}}{\alpha'} &= \frac{1}{2}\nabla^2\Phi - \partial_\mu\Phi\partial^\mu\Phi + \frac{1}{4}|H|^2 + \partial_\mu\Phi\xi^\mu\,. \end{aligned} \tag{4.8}$$

The dots on the left hand side denote derivatives with respect to the logarithm of the renormalization scale. The vector ξ performs small spacetime diffeomorphisms along with RG flow [6], allowing, for example, the imposition of a gauge-fixing condition on the flow. Choosing $\xi_\mu \to \partial_\mu\Phi + \xi_\mu$ eliminates the dilaton [6] from the first two equations in (4.8). In order to determine the flow of the dilaton, however, one needs all the other fields (this illustrates the general principle explained in section 3.7). Since our problem is effectively three dimensional is possible to express the degree of freedom in the antisymmetric tensor $B_{\mu\nu}$ in terms of a scalar by defining $H_{ry\phi} = \epsilon_{ry\phi}\psi$. The RG flow equations reduce to

$$\dot{G}_{\mu\nu} = -R_{\mu\nu} + \frac{1}{2}G_{\mu\nu}\psi^2 + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu\,,$$

$$
\begin{aligned}
\dot{\psi} &= \frac{1}{2}\nabla^2\psi + \frac{1}{2}\psi R - \frac{3}{4}\psi^3 + \partial_\mu\psi\xi^\mu\,, \\
\dot{\Phi} &= \frac{1}{2}\nabla^2\Phi + \frac{1}{4}\psi^2 + \partial_\mu\Phi\xi^\mu\,.
\end{aligned}
\tag{4.9}
$$

In order to study these equations we imposed a gauge in which the metric was diagonal with $g_{\phi\phi}$ fixed, which, given the symmetries of the problem, leaves no remaining coordinate freedom. Our simulations initially reproduced the linear evolution of the normal mode, with the predicted eigenvalue, but upon entering the nonlinear regime of evolution quickly ran into a singularity. However, we are unsure whether the singularity that we saw in our numerics was that described the previous subsection, or simply than a prosaic coordinate singularity due to an unlucky choice of gauge. We present some additional details of these numerical simulations in appendix B.

5. Discussion

In this paper we have studied the compactification of type-II string theories on twisted circles and the condensation of tachyons in this system. We have argued that the final endpoint of tachyon condensation in these models is type-II theory on an untwisted circle. In particular, if our analysis can be trusted in the limit that the system becomes type-0A(B) theory, then it would predict that the endpoint of tachyon condensation in those theories is type-IIB(A) theory in flat space.

While we believe that the arguments of this paper have made a strong case for this conclusion, our arguments have been indirect and several questions remain open. For instance we have not been able to follow the flow of the dilaton, in the passage from twisted to ordinary circles, and so cannot predict its final value. It would certainly be useful to understand the tachyon condensation process in detail. It is thus especially interesting that in the zero-slope limit described in section 4.1 behind, tachyon condensation may be analysed in supergravity. A detailed (perhaps numerical) study of the supergravity evolution might prove very illuminating.

The results of this paper fall into the simple pattern observed in other recent studies of tachyon condensation. In the situations studied so far, it appears that closed string tachyons generically represents the instability of an excited state to decay into a supersymmetric ground state of M theory. It would be fascinating to learn if this more generally true in string theories with tachyons and in particular where the bosonic string fits into this story.

An intriguing feature of our analysis is the appearance of an auxiliary spacetime dimension that could be interpreted as a scale. This is reminiscent of the holographic nature of the AdS/CFT duality, and in particular the duality of little string theories to linear dilaton string theories. In fact, the auxiliary CFT constructed in section 3 of this paper is closely related to little string theories compactified on a twisted circle.[14] It would be interesting to understand these connections better. It would also be very interesting to explore the applications of our results to the Hagedorn transition in finite temperature string theory.

[14]We would like to thank J. Maldacena for bringing this to our attention.

Apart from the perturbative instabilities, type-II theories on twisted circles also have nonperturbative instabilities which correspond to the nucleation of branes in a strong background field. This is reminiscent of the situation in brane-antibrane systems where for large brane separation the open string tachyons disappear [49, 50] and are replaced by a nonperturbative bounce eating up the branes [51, 52]. In [17] it was conjectured that the ten dimensional R-R Melvin flux 7-brane can decay to supersymmetric type IIA. In this paper we analyzed related systems where the flux is in the NS-NS sector and can therefore be analyzed in string perturbation theory. Even in this case there are nonperturbative instabilities, and the results of [53] seem to indicate that perturbative tachyon condensation and nucleation of KK-branes have the same endpoint.

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A. The tachyon in supergravity

In section 2 it was shown that tachyons for a string on twisted circles have winding number $w = \pm k$ and spin $j_l = \pm 1, j_r = \mp 1$. The zero modes are replaced by a set of creation and annihilation operators of a two dimensional harmonic oscillator. The tachyon will be in the ground state wave function of this oscillator. Hence the linear fluctuations of the graviton and antisymmetric tensor associated with the $w = \pm k$ tachyon is given by

$$h_{rr} = e^{-kqr^2/2}\,, \qquad h_{\phi\phi} = r^2 e^{-kqr^2/2}\,, \qquad b_{r\phi} = \pm i r e^{-kqr^2/2}\,. \tag{A.1}$$

There is a subtlety in the identification of the polarization of the vertex operator and the fluctuations of the (flat) supergravity background. In string perturbation theory the graviton and dilaton satisfy a generalized de Donder gauge condition

$$\partial_\mu h^\mu_\nu - \frac{1}{2}\partial_\nu h^\mu_\mu + 2\partial_\nu \Phi = 0\,. \tag{A.2}$$

Since we do not have any momentum in the r, ϕ directions this condition can be satisfied if $\Phi = h^\mu_\mu/4$; hence in addition to (A.1) the tachyon wave functions also contains a dilaton fluctuation given by

$$\delta\Phi = \frac{1}{2} e^{-kqr^2/2} \,. \tag{A.3}$$

The physical tachyon is given by the real combination of the $w = +k$ and $w = -k$ wave functions of the tachyon. There are thus two tachyon states, because we are on a circle, as $R \to 0$ (or $R' \to \infty$ in the T-dual picture) the two states correspond to positive or negative momentum.

The r, ϕ coordinates are flat but have nontrivial identifications, it is therefore useful to express the the polarizations in the single valued (untwisted coordinates), using $\tilde{\phi} = \phi + qy$.

$$\begin{aligned}
h_{yy} &= q^2 r^2 e^{-kqr^2/2} \,, & h_{rr} &= e^{-kqr^2/2} \,, \\
h_{y\tilde{\phi}} &= q r^2 e^{-kqr^2/2} \,, & h_{\tilde{\phi}\tilde{\phi}} &= r^2 e^{-kqr^2/2} \,, \\
b_{r\tilde{\phi}} &= i r e^{-kqr^2/2} \,, & b_{ry} &= \pm i q r e^{-kqr^2/2} \,, \\
\delta\Phi &= \frac{1}{2} e^{-kqr^2/2} \,.
\end{aligned} \tag{A.4}$$

These are small fluctuations around the background of the twisted circle. We now want to perform a T-duality along the y-circle. The winding mode $w = \pm k$ is turned into a momentum mode. The background and the small fluctuations can be obtained using the standard Buscher duality rules [54] and expanding to order linear order in the small fluctuations. The linearized fluctuations are given by

$$\begin{aligned}
h_{yy} &= -\frac{q^2 r^2}{(1+q^2r^2)^2} e^{-kqr^2/2} e^{\pm iky} \,, & h_{rr} &= e^{-kqr^2/2} e^{\pm iky} \,, \\
h_{yr} &= \mp i \frac{qr}{1+q^2r^2} e^{-kqr^2/2} e^{\pm iky} \,, & \tilde{h}_{\tilde{\phi}\tilde{\phi}} &= \frac{r^2}{(1+q^2r^2)^2} e^{-kqr^2/2} e^{\pm iky} \,, \\
\tilde{h}_{y\tilde{\phi}} &= 0 \,, & \tilde{h}_{\tilde{\phi}r} &= 0 \,, \qquad b_{yr} = 0 \,, \\
\tilde{b}_{r\tilde{\phi}} &= \pm i \frac{r}{1+q^2r^2} e^{-kqr^2/2} e^{\pm iky} \,, & \tilde{b}_{y\tilde{\phi}} &= \frac{qr^2}{(1+q^2r^2)^2} e^{-kqr^2/2} e^{\pm iky} \,, \\
\delta\Phi &= \frac{1}{2} \frac{1}{1+q^2r^2} e^{-kqr^2/2} e^{\pm iky} \,.
\end{aligned} \tag{A.5}$$

It is easy to check that (A.5) corresponds a normal mode of the linearized equations of motion growing like e^{mt} where m is the mass of the tachyon $m = \sqrt{2kq - k^2}$, however it would be a formidable task to find (A.5) without the help of T-duality.

The tachyon wave function is localized near the center of the fluxbrane in a region $r < 1/|kq|$, note that this region becomes larger for smaller tachyon momenta k. It is interesting that the tachyon has nonzero momentum in the y direction, since one naively would expect that adding momentum raises the mass of a state. The condensation of the tachyon will therefore break the translation invariance along the y circle of the background.

B. Details of the numerics

We performed a numerical analysis of the supergravity RG flow equation by fixing to a particular gauge. After eliminating the dilaton from the flow equations for the other fields (as described in section 4). We simplify the general flow equations

$$
\begin{aligned}
\dot{g}_{ij} &= -R_{ij} + \partial_i\sigma\partial_j\sigma + \nabla_i\partial_j\sigma + \frac{1}{2}g_{ij}\psi^2 + \nabla_i\xi'_j + \nabla_j\xi'_i\,,\\
\dot{\sigma} &= \frac{1}{2}\nabla^2\sigma + \frac{1}{2}\partial_i\sigma\partial^i\sigma + \frac{1}{4}\psi^2 + \xi'^i\partial_i\sigma\,,\\
\dot{\psi} &= \frac{1}{2}\nabla^2\psi + \frac{1}{2}\partial_i\psi\partial^i\sigma + \psi\left(\frac{1}{2}R - \partial_i\sigma\partial^i\sigma - \nabla^2\sigma\right) - \frac{3}{4}\psi^3 + \xi'^i\partial_i\psi\,,\\
\dot{\Phi} &= \frac{1}{2}\nabla^2\Phi + \frac{1}{2}\partial_i\Phi\partial^i\sigma + \frac{1}{4}\psi^2 + \xi'^i\partial_i\Phi\,.
\end{aligned}
\tag{B.1}
$$

by choosing ξ so that the flow respects the gauge (from this point we set $q = 1$ for simplicity):

$$
ds^2 = e^{2\eta}dy^2 + e^{2\alpha}dr^2 + e^{2\sigma}d\varphi^2\,, \qquad \sigma = \ln\frac{r}{\sqrt{1+r^2}}\,. \tag{B.2}
$$

In order for evolution to respect this gauge, ξ must be chosen such that $\dot{\sigma} = \dot{g}_{ry} = 0$. We may solve these equations for ξ':

$$
\begin{aligned}
\xi'^r &= \frac{1}{2}e^{-2\alpha}\left(\partial_r\alpha - \partial_r\eta + \frac{3r}{1+r^2}\right) - \frac{1}{4}r(1+r^2)\psi^2\,,\\
\xi'^y(r') &= -\int_{r'}^{\infty} dr\, e^{-2\eta}\left(\frac{\partial_y\alpha}{r(1+r^2)} - e^{2\alpha}\partial_y\xi'^r\right).
\end{aligned}
\tag{B.3}
$$

(Note that the integral in ξ'^y should be done from infinity in order to leave the asymptotic region invariant.) The RG flow equations in this gauge then become:

$$
\begin{aligned}
\dot{\eta} &= -\frac{1}{4}R + \frac{e^{-2\alpha}\partial_r\eta}{2r(1+r^2)} + \frac{1}{4}\psi^2 + \partial_y\xi'^y + \xi'^y\partial_y\eta + \xi'^r\partial_r\eta\,,\\
\dot{\alpha} &= -\frac{1}{4}R - \frac{1}{2}e^{-2\alpha}\left(\frac{3}{(1+r^2)^2} + \frac{\partial_r\alpha}{r(1+r^2)}\right) + \frac{1}{4}\psi^2 + \partial_r\xi'^r + \xi'^y\partial_y\alpha + \xi'^r\partial_r\alpha\,,\\
\dot{\psi} &= \frac{1}{2}e^{-2\eta}\left(\partial_y^2\psi + (\partial_y\alpha - \partial_y\eta)\partial_y\psi\right) + \frac{1}{2}e^{-2\alpha}\left(\partial_r^2\psi + \left(\partial_r\eta - \partial_r\alpha + \frac{1}{r(1+r^2)}\right)\partial_r\psi\right) +\\
&\quad + \psi\left(\frac{1}{2}R + e^{-2\alpha}\left(\frac{3}{(1+r^2)^2} + \frac{\partial_r\alpha - \partial_r\eta}{r(1+r^2)}\right) - \frac{3}{4}\psi^2\right) + \xi'^r\partial_r\psi + \xi'^y\partial_y\psi\,,
\end{aligned}
\tag{B.4}
$$

where the Ricci scalar R is given by

$$
R = 2e^{-2\eta}\left(\partial_y\alpha\partial_y\eta - (\partial_y\alpha)^2 - \partial_y^2\alpha\right) + 2e^{-2\alpha}\left(\partial_r\alpha\partial_r\eta - (\partial_r\eta)^2 - \partial_r^2\eta\right). \tag{B.5}
$$

We numerically solved the gauged fixed equations (B.4) using an explicit time stepping algorithm, solving for the gauge fixing constraints (B.3) at each time step and imposing the appropriate boundary conditions for $r = 0$ and a large r. The initial conditions were giving by the background (4.6) plus a small tachyonic perturbation (A.5).

C. Twisted circles on the worldsheet

C.1 CFT description

We will review the mode expansions of twisted circles on the world sheet. We use Euclidean world sheet conventions which makes it convenient to apply conformal field theory methods. In the discussion below we will set $\alpha' = 2$.

The bosonic part of the world sheet action is given by

$$S = \frac{1}{4\pi}\int d^2z \partial X^\mu \bar{\partial} X_\mu \,. \tag{C.1}$$

Let us restrict our attention to the coordinates of the 2-plane which have twisted boundary conditions in type-IIA. In terms of free fields $X^+ = X^1 + iX^2$ and $X^- = X^1 - iX^2$ the twisted circle background satsifies the following boundary conditions

$$\begin{aligned} X^+(e^{2\pi i}z, e^{-2\pi i}\bar{z}) &= e^{2\pi i\nu} X^+(z,\bar{z})\,, \\ X^-(e^{2\pi i}z, e^{-2\pi i}\bar{z}) &= e^{-2\pi i\nu} X^-(z,\bar{z})\,. \end{aligned} \tag{C.2}$$

Here $\nu = w\zeta = wqR$ with $0 < \nu < 1$. The mode expansions for these fields are given by

$$X^+(z,\bar{z}) = i\sqrt{2}\sum_n \left(\frac{1}{n-\nu}\frac{\alpha^+_{n-\nu}}{z^{n-\nu}} + \frac{1}{n+\nu}\frac{\tilde{\alpha}^+_{n+\nu}}{\bar{z}^{n+\nu}} \right), \tag{C.3}$$

$$X^-(z,\bar{z}) = i\sqrt{2}\sum_n \left(\frac{1}{n+\nu}\frac{\alpha^-_{n+\nu}}{z^{n+\nu}} + \frac{1}{n-\nu}\frac{\tilde{\alpha}^-_{n-\nu}}{\bar{z}^{n-\nu}} \right). \tag{C.4}$$

The canonical commutation relations for the modes are given by

$$[\alpha^+_{n-\nu}, \alpha^-_{m+\nu}] = (n-\nu)\delta(n+m), [\tilde{\alpha}^+_{n+\nu}, \tilde{\alpha}^-_{m-\nu}] = (n+\nu)\delta(n+m)\,. \tag{C.5}$$

Substituting the mode expansions (C.4) in the definition of the zero modes L_0 and $\bar{L}_0$ of the stress energy tensor and using the commutations relations (C.5) we obtain

$$\begin{aligned} L_0 &= \sum_{n=1}^{\infty}(\alpha^-_{-n+\nu}\alpha^+_{n-\nu} + \alpha^+_{-n-\nu}\alpha^-_{n+\nu}) + \alpha^+_{-\nu}\alpha^-_{\nu} - \frac{1}{12} + \frac{1}{2}\nu(1-\nu)\,, \\ \bar{L}_0 &= \sum_{n=1}^{\infty}(\tilde{\alpha}^+_{-n+\nu}\tilde{\alpha}^-_{n-\nu} + \tilde{\alpha}^-_{-n-\nu}\tilde{\alpha}^+_{n+\nu}) + \tilde{\alpha}^-_{-\nu}\tilde{\alpha}^+_{\nu} - \frac{1}{12} + \frac{1}{2}\nu(1-\nu)\,. \end{aligned} \tag{C.6}$$

Here L_0, $\bar{L}_0$ is written with the fractional moding such that the linear term which occurs with $j_L - j_R$ is not present. It can be easily shown that this is the same as (2.2).

The operator which creates the twisted sector out of the vacuum is given by $\sigma_\nu(z,\bar{z})$. It is defined by the following operator product expansions [55].

$$\begin{aligned} \partial X^+(z)\sigma_\nu(w,\bar{w}) &= \frac{\tau_\nu(w,\bar{w})}{(z-w)^{1-\nu}}\,, & \partial X^-(z)\sigma_\nu(w,\bar{w}) &= \frac{\tau'_\nu(w,\bar{w})}{(z-w)^{\nu}}\,, \\ \bar{\partial} X^+(\bar{z})\sigma_\nu(w,\bar{w}) &= \frac{\tilde{\tau}'_\nu(w,\bar{w})}{(\bar{z}-\bar{w})^{\nu}}\,, & \bar{\partial} X^-(\bar{z})\sigma_\nu(w,\bar{w}) &= \frac{\tilde{\tau}_\nu(w,\bar{w})}{(\bar{z}-\bar{w})^{1-\nu}}\,. \end{aligned} \tag{C.7}$$

These equations also define the excited twist operators τ's. There are similar operator product expansions for the anti-twist operator $\sigma_{-\nu}(z,\bar{z})$. The conformal dimension of the twist operator σ_ν or the anti-twist operator $\sigma_{-\nu}$ is $h = \bar{h} = \nu(1-\nu)/2$. Note that this is the contribution of the zero point energy due to the fractional moding in (C.6).

The world sheet action for the fermions is given by

$$S = \frac{1}{4\pi}\int d^2z \left(\bar{\partial}\psi^\mu\psi_\mu - \tilde{\psi}^\mu \partial\tilde{\psi}_\mu \right). \tag{C.8}$$

Using the suspersymmetric variation on the (C.4) we find the modes expansion of the superpartners of X^+ and X^-.

$$\begin{aligned}
\psi^+(z) &= i\sqrt{2}\sum_r \frac{\psi^+_{r-\nu}}{z^{r-\nu+1/2}}\,, \qquad & \psi^-(z) &= i\sqrt{2}\sum_r \frac{\psi^-_{r+\nu}}{z^{r+\nu+1/2}}\,,\\
\tilde{\psi}^+(\bar{z}) &= i\sqrt{2}\sum_r \frac{\tilde{\psi}^+_{r+\nu}}{\bar{z}^{r+\nu+1/2}}\,, \qquad & \tilde{\psi}^-(\bar{z}) &= i\sqrt{2}\sum_r \frac{\tilde{\psi}^-_{r-\nu}}{\bar{z}^{r-\nu+1/2}}\,.
\end{aligned} \tag{C.9}$$

Here r is half integral in the Neveu-Schwarz sector and integral in the Ramond sector. We will focus on the Neveu-Schwarz sector therefore for the rest of the discussion r is half integral. The anti-commutation relations are given by

$$\begin{aligned}
\{\psi^+_{r-\nu}, \psi^-_{s+\nu}\} &= \delta(r+s)\,,\\
\{\tilde{\psi}^+_{r+\nu}, \tilde{\psi}^-_{s-\nu}\} &= \delta(r+s)\,.
\end{aligned} \tag{C.10}$$

Substituting the expansion (C.9) for the zero mode of the stress energy tensor and using (C.10) we obtain

$$\begin{aligned}
L_0 &= \sum_{r>0}(r-\nu)\psi^-_{-r+\nu}\psi^+_{r-\nu} + \sum_{r>0}(r+\nu)\psi^+_{-r-\nu}\psi^-_{r+\nu} - \frac{1}{24} + \frac{\nu^2}{2}\,,\\
\bar{L}_0 &= \sum_{r>0}(r-\nu)\tilde{\psi}^+_{-r+\nu}\tilde{\psi}^-_{r-\nu} + \sum_{r>0}(r+\nu)\tilde{\psi}^-_{-r-\nu}\tilde{\psi}^+_{r+\nu} - \frac{1}{24} + \frac{\nu^2}{2}\,.
\end{aligned} \tag{C.11}$$

Here to make our discussion less cumbersome we have assumed $0 < \nu < 1/2$. The summation over r in the above expressions run over the positive half integers.

To construct the fermionic twist operators we first bosonize the fermions by defining

$$\begin{aligned}
\psi^+(z) &= i\sqrt{2}e^{iH(z)}\,, \qquad & \psi^-(z) &= i\sqrt{2}e^{-iH(z)}\,,\\
\tilde{\psi}^+(\bar{z}) &= i\sqrt{2}e^{i\tilde{H}(\bar{z})}\,, \qquad & \tilde{\psi}^-(\bar{z}) &= i\sqrt{2}e^{-i\tilde{H}(\bar{z})}\,.
\end{aligned} \tag{C.12}$$

The twist operator is given by $e^{i\nu(H(z)-\tilde{H}(\bar{z}))}$. Note that the dimension of this twist operator is the same as the zero point energy of the fermions due to the fractional modding in (C.11). From (C.6) and (C.11) it is easy to see that the full zero point energy including the remaining 3 complex untwisted coordinates is given by $\nu/2 - 1/2$.

Due to the twisted circle boundary conditions (1.1) the zero modes of the Y coordinate are quantized as follows.

$$\begin{aligned}
p_L + p_R &= 2\left(\frac{n}{R} - \frac{(j_L + j_R)\nu}{R}\right),\\
p_L - p_R &= Rw.
\end{aligned} \tag{C.13}$$

The L_0 and $\bar{L}_0$ contribution from these zero modes is $p_L^2/2$ and $p_R^2/2$ respectively.

C.2 The tachyon vertex operator

We now construct the vertex operator corresponding to the tachyon which survives in the zero slope limit. Consider the following state over the twisted vacuum

$$\psi^-_{-1/2+\nu}\tilde{\psi}^+_{-1/2+\nu}|\mathrm{Twist}, w\rangle\,. \tag{C.14}$$

Let us write this as a vertex operator in the $(-1,-1)$ picture,

$$\int d^2z e^{-\phi(z)}e^{-\tilde{\phi}(\bar{z})}\sigma_\nu(z,\bar{z})e^{i(\nu-1)H(z)}e^{-i(\nu-1)\tilde{H}(\bar{z})}e^{i\frac{wR}{2}(Y(z)-\tilde{Y}(\bar{z}))}\,. \tag{C.15}$$

Here $\phi(z)$ and $\tilde{\phi}(z)$ are the superconformal ghosts. This state has $j_L + j_R = 0$. Note that the conformal dimension of this operator is given by

$$L_0 = -\frac{\nu}{2} + \frac{w^2R^2}{8}\,, \qquad \bar{L}_0 = -\frac{\nu}{2} + \frac{w^2R^2}{8}\,. \tag{C.16}$$

This gives the following mass for the field

$$M^2 = \frac{1}{4}(wR - 2q)^2 - q^2\,, \tag{C.17}$$

where $\nu = w\zeta = wqR$. This agrees with the mass formula (4.3). Note also there is a tachyon from the anti-twist operator corresponding to the state in the opposite winding sector. We write down its vertex operator,

$$\int d^2z e^{-\phi(z)}e^{-\tilde{\phi}(\bar{z})}\sigma_{-\nu}(z,\bar{z})e^{-i(\nu-1)H(z)}e^{+i(\nu-1)\tilde{H}(\bar{z})}e^{-i\frac{wR}{2}(Y(z)-\tilde{Y}(\bar{z}))}\,. \tag{C.18}$$

This state also has the same mass given by (C.17).

D. The tachyon potential near the critical radius

In this appendix we evaluate the tachyon potential in the zero slope limit for a tachyon close to marginality. We use strategy developed in [56] for evaluating the tachyon potential. We first evaluate the 4-pt tachyon on shell amplitude. By factorizing this amplitude along various channels we can extract the cubic couplings involving two tachyons and other fields which survives the zero slope limit. Subtracting the contribution of the massless exchanges in the 4-pt tachyon amplitude we can extract the leading contact quartic interaction term involving the tachyons.

D.1 The 4-point tachyon on-shell amplitude

The tachyon vertex operator in the $(-1,-1)$ picture is given by

$$\mathcal{T} = e^{-\phi(z)}e^{-\tilde{\phi}(\bar{z})}\sigma_\nu(z,\bar{z})e^{i(\nu-1)H(z)}e^{-i(\nu-1)\tilde{H}(\bar{z})}e^{i\frac{wR}{2}(Y(z)-\tilde{Y}(\bar{z}))}e^{ik\cdot x(z,\bar{z})}\,. \tag{D.1}$$

For the vertex operator to be on shell $-k^2 = M^2$ where M^2 is given in (C.17). Similarly the vertex operator for the complex conjugate tachyon is given by

$$\bar{\mathcal{T}} = e^{-\phi(z)}e^{-\tilde{\phi}(\bar{z})}\sigma_{-\nu}(z,\bar{z})e^{-i(\nu-1)H(z)}e^{+i(\nu-1)\tilde{H}(\bar{z})}e^{-i\frac{wR}{2}(Y(z)-\tilde{Y}(\bar{z}))}e^{ik\cdot x(z,\bar{z})}\,. \tag{D.2}$$

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To compute the four point amplitude we need to find the tachyon vertex operator in the $(0,0)$ picture. This is done by the action of the picture changing operator $e^{\phi}T_F e^{\tilde{\phi}}\tilde{T}_F$ on the operators in (D.1) and (D.2). T_F is the supercharge given by

$$T_F = -\frac{1}{4}(\partial X^+\psi^- + \partial X^-\psi^+) - \frac{1}{2}(\partial Y\psi^Y + \partial x^\mu\psi_\mu)\,. \tag{D.3}$$

Here ψ^Y and ψ^μ are superpartners of Y and x^μ respectively.

The four point scattering amplitude is given by

$$\mathcal{A} = 2\pi\int d^2x\langle c(\infty)\tilde{c}(\infty)\bar{\mathcal{T}}(\infty,\infty)c(1)\tilde{c}(1)e^{\phi}T_F e^{\bar{\phi}}\bar{T}_F\mathcal{T}(1)\bar{\mathcal{T}}(x,\bar{x})c(0)\tilde{c}(0)e^{\phi}T_F e^{\bar{\phi}}\bar{T}_F\mathcal{T}(0)\rangle\,. \tag{D.4}$$

One can show that the excited twist operators do not contribute in this correlation function. Let k_4, k_3, k_2, k_1 be the space time momentum of the tachyon operators located at ∞, 1, x, 0 respectively. Evaluating all the correlation functions we get

$$\mathcal{A} = 2\pi\int d^2x\left(k_3\cdot k_1 + \frac{w^2R^2}{4}\right)^2|1-x|^{2k_3\cdot k_2-4h}|x|^{2k_2\cdot k_1-4h}I(x,\bar{x})\,, \tag{D.5}$$

where

$$h = \frac{1-\nu}{2} + \frac{w^2R^2}{8} \tag{D.6}$$

and $I(x,\bar{x})$ is given by

$$I(x,\bar{x}) = \left(F(x)F(1-\bar{x}) + F(\bar{x})F(1-x)\right)^{-1}\,, \tag{D.7}$$

where $F(x)$ is the hypergeometric function $F(\nu,1-\nu,1;x)$.[15]

D.2 Cubic couplings of the tachyon

We find the cubic couplings of the tachyon by factorizing the four-point amplitude into its various channels. To extract the $s=(k_1+k_2)^2$, channel process look at the limit $x\to 0$. Using the following property

$$\lim_{x\to 0} I(x,\bar{x}) = 1 \tag{D.8}$$

and integrating around $x=0$ in (D.5) we get

$$\mathcal{A} = 4\pi^2\left(k_3\cdot k_1 + \frac{w^2R^2}{4}\right)^2\frac{1}{s}\,. \tag{D.9}$$

Similarly the amplitude factorized along the $t=(k_3+k_2)^2$, channel is given by

$$\mathcal{A} = 4\pi^2\left(k_3\cdot k_1 + \frac{w^2R^2}{4}\right)^2\frac{1}{t}\,. \tag{D.10}$$

Both these terms arise from exchange of the graviton, the antisymmetric tensor, the dilaton and the scalars that resulted from the compactification of the graviton and the

[15] There is no lattice sum involved as the directions X^+ and X^- are non-compact.

antisymmetric tensor. From (D.9) and (D.10) the cubic couplings of the tachyon with the massless fields can be read out. The zero momentum tachyon couples only to the fluctuation of the g_{yy} component of the metric. Therefore the cubic coupling is given by

$$2\pi \int d^7x \frac{w^2R^2}{4} T\bar{T}\varphi\,, \tag{D.11}$$

here T stands for the tachyon and φ is the fluctuation of the g_{yy} component of the metric. Note that the OPE of a tachyon and its complex conjugate results only in the untwisted sector due to charge conservation. Restricting the factorization only to the fields which survive the zero slope limit we obtain only the massless fields discussed above.

It is possible that there might be cubic couplings of two tachyons with $w = 1$ to a state of winding $w = 2$ with mass given by (C.17). We show that this is not possible for $2\nu < 1$, which holds in the zero slope limit. We see that two tachyons with $w = 1$ couple only to states with masses of the order of string scale.

The cubic couplings of two tachyons with $w = 1$ to a state with $w = 2$ can be extracted from the $u = (k_2 + k_4)^2$, channel. We can extract the leading exchange in the u channel by taking the limit $x \to \infty$. The following identities are helpful in obtaining this limit.

$$F(\nu, 1-\nu, 1; x) = (1-x)^{-\nu} F\left(\nu, \nu, 1, \frac{x}{x-1}\right)\,, \tag{D.12}$$

$$F(\nu, \nu, 1; 1) = \frac{\Gamma(1-2\nu)}{\Gamma^2(1-\nu))} \qquad (1 - 2\nu > 0)\,. \tag{D.13}$$

The leading $u-$channel exchange in the four-point amplitude is given by

$$4\pi^2 \left(k_3 \cdot k_1 + \frac{w^2R^2}{4}\right)^2 \frac{\Gamma^4(1-\nu)}{2\cos\pi\nu(\Gamma^2(1-2\nu))} \frac{1}{u + 2 - 6\nu + w^2R^2}\,. \tag{D.14}$$

Note that the leading pole is due to the exchange of a particle of string mass. This state can be created by the following vertex operator in the $(0,0)$ picture.

$$\int d^2z \sigma_{2\nu}(z,\bar{z}) e^{2i(\nu-1)H(z)} e^{-2i(\nu-1)\tilde{H}(\bar{z})} e^{wR(Y(z)-\tilde{Y}(\bar{z}))}\,. \tag{D.15}$$

Thus we conclude that the zero momentum tachyon has cubic couplings only with the metric fluctuation of g_{yy}, φ in the zero slope limit.

D.3 Quartic coupling of the tachyon

We compute the quartic coupling of the tachyon by subtracting the s channel and t channel poles from the four-point amplitude. As we are interested in the tachyon potential in the zero slope limit it is consistent to set $\nu = 0$ to obtain the leading term in the four-point amplitude. We are also expanding the tachyon potential when the tachyon is almost marginal, thus all the momenta can be set to zero in the on-shell amplitude. Setting $\nu = 0$ using marginality of the tachyon in (D.5) we obtain

$$\mathcal{A} = 2\pi^2 \left(k_3 \cdot k_1 + \frac{w^2R^2}{4}\right)^2 \frac{\Gamma(t/2)\Gamma(s/2)\Gamma(u/2+1)}{\Gamma(-u/2)\Gamma(1-s/2)\Gamma(1-t/2)}\,. \tag{D.16}$$

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Subtracting the s and t channel poles and setting the momenta to zero we obtain

$$-16\pi^2\gamma\frac{w^4R^4}{16}, \tag{D.17}$$

where $\gamma = .5772$, the Euler-Mascheroni constant. Thus the zero momentum quartic coupling for a marginal tachyon is given by

$$-16\pi^2\gamma\int d^7x\frac{w^4R^4}{16}(T\bar{T})^2\,. \tag{D.18}$$

E. RG flows near the critical radius

As we have discussed in section 4.3, the fluxbrane ceases to be unstable upon compactifying y (see (4.6)) on a circle of radius $R' < R'_{\rm cr} = 1/2q$ as its lowest-momentum tachyon mode is actually massive. Consequently, the fluxbrane undergoes a 'phase transition' at $R = R'_{\rm cr}$.

The tachyon at R marginally greater than $R'_{\rm cr}$ is almost massless. It is of interest to ask: Does its condensation an endpoint to tachyon condensation 'near' the inital unstable flux brane? Relatedly, just below the critical radius, is the flux brane stable to small but finite perturbations. The picture developed in section 3 of this paper would suggest the answers in the negative to these questions. We independently confirm these answers in this appendix. Our analysis uses the results derived in appendix D.

In the neighborhood of the critical radius, and for the small values of the tachyon, it is possible to argue (see appendix D) that tachyon dynamics is governed by an effective potential that contains only two modes; the zero momentum tachyon T and φ, the zero momentum mode corresponding to the fluctuation of the metric component g_{yy}. It is a modulus of the theory. Physically this mode corresponds to change in the radius R' and the twist parametrized by q. This change is given by[16]

$$\begin{aligned} R' &\to \frac{R'}{\sqrt{1+4\pi\varphi}}\,, \\ q &\to \frac{q}{\sqrt{1+4\pi\varphi}}\,. \end{aligned} \tag{E.1}$$

It is easy to see that the total flux which is proportional to the ratio q/R' does not change under a change in φ, reflecting the fact that φ is a modulus. From the three-point and the four-point couplings of the tachyon we obtain the following potential

$$V = -\frac{1}{2}\left(2qk - k^2\right)T\bar{T} + 2\pi k^2 T\bar{T}\varphi - 16\pi^2\gamma k^4(T\bar{T})^2 \tag{E.2}$$

(γ is the Euler-Mascheroni constant). Here k is the momentum of the tachyon in the y direction, and it is assumed that the tachyon is almost marginal, i.e. $k = 1/R' \approx 2q$. The coefficients of this potential are computed in appendix D using conformal field theory

[16]The factor 4π occurs in relating the fluctuation coefficient in the vertex operator to the fluctuations of the metric, see eq. (6.6.18) of [37].

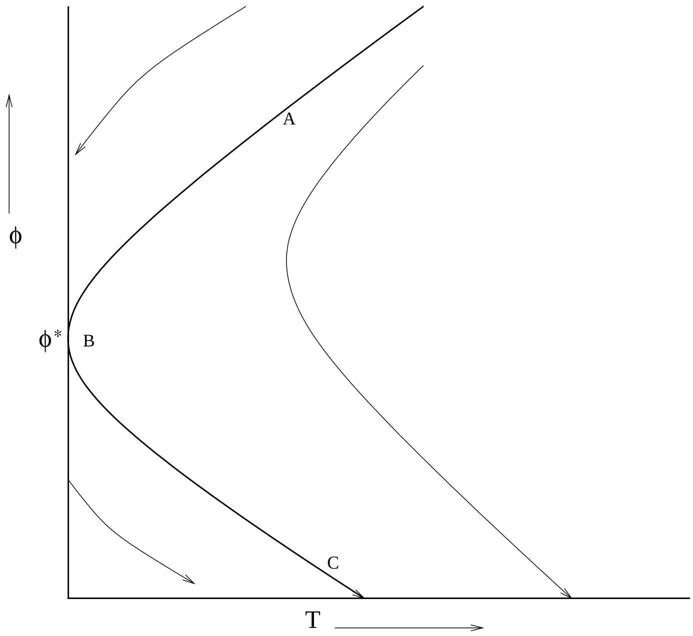

Figure 1: Sketch of RG flow in the $|T|$-φ plane. The trajectory ABC is the separatrix.

techniques. From this potential it is easy to evaluate the beta functions for $|T|$ and for φ. They are given by

$$\begin{aligned} \dot{|T|} &= (2qk - k^2)|T| - 4\pi k^2|T|\varphi + 64\pi^2\gamma k^4|T|^3\,, \\ \dot{\varphi} &= -2\pi k^2|T|^2\,. \end{aligned} \tag{E.3}$$

The phase portraits for these flows are plotted below. Note that this phase diagram has no nontrivial fixed points (i.e. fixed points with $|T| \neq 0$) irrespective of whether the tachyon mass is positive or negative. In fact the only fixed points in this approximation are on the line $|T| = 0$. These fixed points are stable for $\varphi > \varphi^*$ where

$$\varphi^* = \frac{2q - k}{4\pi k}\,. \tag{E.4}$$

Thus φ^* is the value of of φ at which the linear effective mass of the tachyon in the first equation of (E.3) vanishes. The separatrix ABC, separates flows that end on line $|T| = 0$ for $\varphi > \varphi^*$ and flows that runaway to regions beyond the approximation of small $|T|$. Notice that a slight perturbation away from the fixed point $\varphi = 0$, $|T| = 0$ takes the flow far away, beyond the approximation of small $|T|$.

We were able to qualitatively confirm these results with our numerical simulations. which clearly indicated that just below the critical radius the system was not stable against finite perturbations.

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